
Overall

Problem 1. Let $u \in C^\infty(\mathbb{R}^2, \mathbb{R})$ be 1-periodic in each variable. Prove that for any 2×2 real matrix A ,

$$\int_{[0,1]^n} \det(D^2u + A) = \det A,$$

where D^2u denotes the Hessian matrix.

Problem 2. Let A and B be bounded invertible operators on a Hilbert space.

- a) Prove that $\sigma(AB) = \sigma(BA)$ where $\sigma(X)$ denotes the spectrum of X .
- b) What can you say if they are not invertible ?